New Approaches to Target Mass Corrections

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1) Variables

Target four-vector: $\boldsymbol{P^{\mu} = p^{\mu} + \frac{1}{2} m_N^2 n^{\mu}}$

Photon four-vector: $\boldsymbol{q^{\mu} = -\xi p^{\mu} + \frac{Q^2}{2\xi} n^{\mu}}$

Parton four-vector: $\boldsymbol{k^{\mu} = x p^{\mu} + \frac{k^2 + k_T^2}{2x} n^{\mu} + k_{T\mu}}$

Bjorken variable: $\boldsymbol{x_B = \frac{Q^2}{2P \cdot q}}$

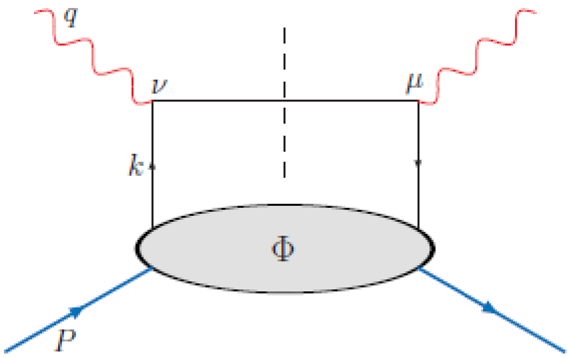
Natchtman variable: $\boldsymbol{\xi = \frac{Q^2}{2p \cdot q}}$

Parton momentum fraction: $\boldsymbol{x = k \cdot n}$

Auxiliary four-vectors:

$$\boldsymbol{p^2 = n^2 = p \cdot k_T = n \cdot k_T = 0}$$
$$\boldsymbol{p \cdot n = 1}$$

The handbag diagram



If the partons are always on-mass shell:

$$\delta[(k+q)^2] = \frac{1}{2P\cdot q}\,\delta\left(-x_B + x + x\xi x_B \frac{m_N^2}{Q^2}\right)$$

with $k_T^2 = 0.$

For a massless target:

$$m_N^2 = 0 \;\Rightarrow x = x_B$$

In general, however:

$$\frac{Q^2}{m_N^2} = \frac{x_B \xi^2}{x_B - \xi} \Rightarrow x = \xi$$

And the parton momentum fraction should be identified with the Nachtmann variable

2) Structure Function with target mass effects

From the OPE (Georgi and Politzer PRD14 (1976) 1829):

$$F_2\left(x_B, Q^2\right) = \frac{\xi^2\left(1 - a^2\xi^2\right)}{(1 + a^2\xi^2)^3} F(\xi) + 6a^2 \frac{\xi^3\left(1 - a^2\xi^2\right)}{(1 + a^2\xi^2)^4} H(\xi) + 12a^4 \frac{\xi^4\left(1 - a^2\xi^2\right)}{(1 + a^2\xi^2)^5} G(\xi)$$

where:

$$a = \frac{m_N}{Q}$$

$$A_n = \int_0^1 dx x^n F(x)$$

$$H(\xi) = \int_\xi^1 d\eta F(\eta)$$

$$G(\xi) = \int_\xi^1 d\eta H(\eta)$$

Parton distribution: $F(x)$
Moments of the parton distribution: A_n

Problem

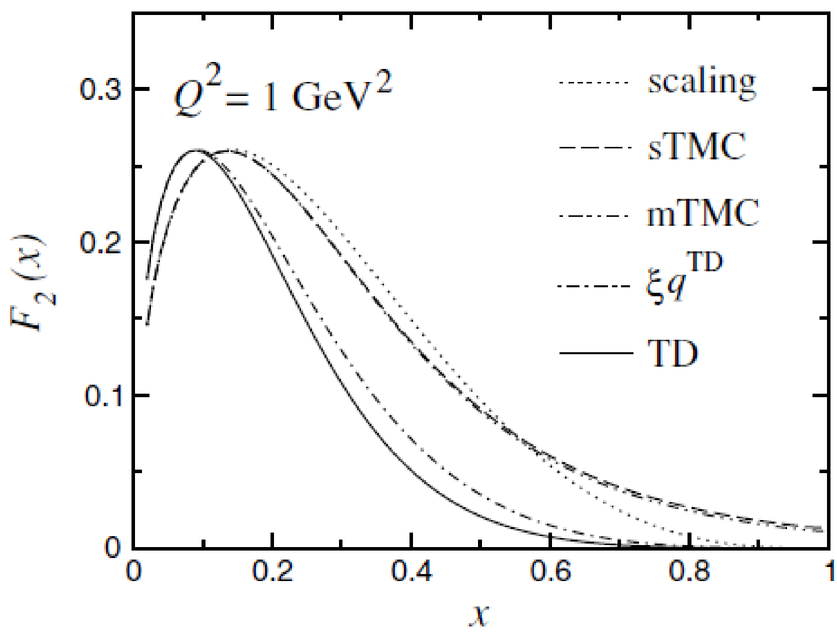
Are the parton distributions defined up to 1 in the presence of a finite target mass?

They should not! The maximum value of the parton momentum fraction is:

$$\xi_0 \equiv \xi(x_B = 1) = \frac{2}{1+\sqrt{1+4a^2}}$$

Which is smaller than 1 for a finite target mass!

Conclusion: we have contributions from an unphysical region



People HOPE that higher twists somehow cancel these contributions…

3) The D'Alesio, Leader and Murgia (DEM) approach – PRD81:036010 (2010)

They follow Ellis, Furmanski, Petronzio (EFP) - NPB212 (1983) 29:

Keep partons on-mass shell, $k^2 = 0$, and retain the transverse momentum

They call this the transverse basis

The hadronic tensor:

$$W^{\mu\nu} \propto \int d^4k \, \delta[(k+q)^2] A$$

Matrix Elements of local operators

However: $\int d^4k \propto \int \frac{dx}{x} \, dk_T^2 \, dk^2$

They introduce the variable:

$$\eta = \frac{2P \cdot k}{m_N^2} = x + \frac{k_T^2}{x m_N^2} = \frac{Q^2}{\xi^2 m_N^2}(Dx - \xi)$$

with $D = 1 + \xi^2 m_N^2 / Q^2$.

It then follows that $\eta \geq \xi$

What about the upper limit for eta?

If one imposes to the lower part of the handbag diagram the following constraint:

$$(P-k)^2 \geq 0 \Rightarrow P^2 - 2P \cdot k \geq 0$$

Or

$$1 - \eta \geq 0 \rightarrow \eta \leq 1$$

Thus:

$$\int dx \rightarrow \int_{\xi}^{1} d\eta$$

Their expressions agree with the results from the OPE

But the problem of the unphysical contribution continues…

4) Steffens and Melnitchouk – PRC73:055202

Define: $\boldsymbol{A_n = \int_0^{y_0} dy y^n F(y)}$

Where $\boldsymbol{y_0}$ is the maximum value physically supported by the distribution $\boldsymbol{F(y)}$.

Repeating the Georgi and Politzer steps, one gets:

$$\boldsymbol{y_0 = \xi_0}$$

and

$$\boldsymbol{F_2\left(x_B, Q^2\right) = \frac{\xi^2\left(1-a^2\xi^2\right)}{(1+a^2\xi^2)^3}F(\xi) + 6a^2\frac{\xi^3\left(1-a^2\xi^2\right)}{(1+a^2\xi^2)^4}H(\xi) + 12a^4\frac{\xi^4\left(1-a^2\xi^2\right)}{(1+a^2\xi^2)^5}G(\xi)}$$

$$\boldsymbol{A_n = \int_0^{\xi_0} dx x^n F(x)}$$

$$\boldsymbol{H(\xi) = \int_\xi^{\xi_0} d\eta F(\eta)}$$

$$\boldsymbol{G(\xi) = \int_\xi^{\xi_0} d\eta H(\eta)}$$

Major problem: moments of parton distributions become Q^2 dependent!

We lose the partonic interpretation

On the other hand, if we let the upper limit reach 1, we include an unphysical region in the calculation…

True Conundrum!

Can we get our result from DEM?

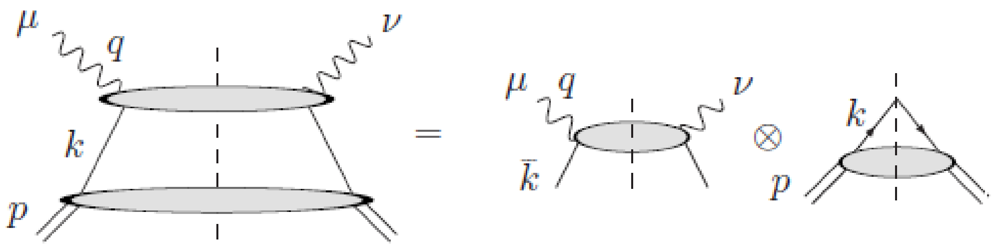
Suppose one imposes the following condition on the lower part of the handbag diagram:

$$(\boldsymbol{P}-\boldsymbol{k}) \geq (\mathbf{1}-\boldsymbol{\xi}_{\mathbf{0}})\boldsymbol{m}_{\boldsymbol{N}}^{\mathbf{2}}$$

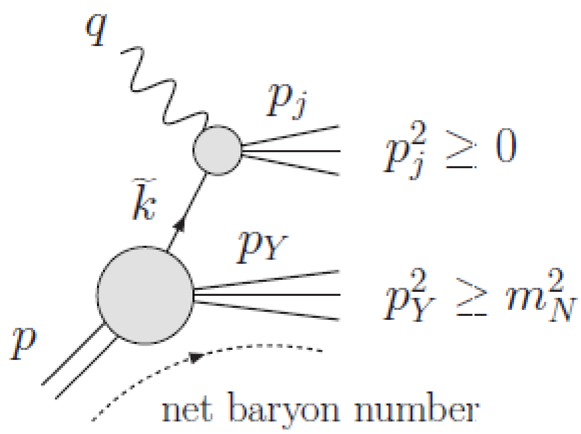
Thus

$$\boldsymbol{\eta} \leq \boldsymbol{\xi}_{\mathbf{0}}$$

5) Accardi and Qui – JHEP 0807:090



Collinear factorization in the impulse approximation



They assume that the relevant contribution comes from the graph where the baryon number flows through the lower part of the graph

$$\mathbf{0 \leq (k+q)^2 \leq (P+q)^2 - m_N^2}$$

Difference to DEM:

a) The parton momentum has no transverse part
b) The parton in the upper part of the handbag diagram can be off-mass shell

Consequence:

$$\xi \leq x \leq \frac{\xi}{x_B}$$

$$F_{T,L}(x_B, Q^2, m_N^2) = \sum_f \int_{\xi}^{\xi/x_B} \frac{dx}{x} h_{f|T,L}\left(\frac{\xi}{x}, Q^2\right) \varphi_f(x, Q^2)$$

In principle, it has the correct support. However, at the tree level:

$$F_T \propto \delta\left(\frac{\xi}{x} - 1\right)$$

and the structure function does not vanish at $x_B = 1$…

They invoke jet mass corrections to solve this problem, but it is a phenomenological fix…

Two additional potential problems:

a) Why should the baryon number flows entirely over the lower part of the handbag diagram?

b) The first moment:

$$\int_0^1 F_{T,L}(x_B, Q^2, m_N^2) dx_B = \sum_f \int_0^{\xi_0} dx \varphi_f(x, Q^2) \int_0^{u(x)} dy h_{f|T,L}(y, Q^2) \frac{1 + a^2x^2y^2}{(1 - a^2x^2y^2)^2}$$

with

$$u(x) = \frac{2}{\xi_0 + \sqrt{\xi_0 + 4a^2x^2}}$$

Cannot separate the first moment in a soft and in a hard part!

Conclusions

1) OPE includes TMC BUT the parton distributions are defined in an unphysical region – HT correction of the problem is a dark box…

2) Partonic approach in the transverse basis reproduces OPE: in its glory and in its failure!

3) Inclusion of the correct physical contribution to the OPE leads to the breakdown of the concept of universal parton distributions…

4) Collinear factorization does not solve any of these problems either

It seems that if target mass is included, one loses the partonic interpretation: no parton distributions with TMC can be really defined!